AESB 2320 Part 2 Exam 19 April 2017

1. How does this differ from section 9.2 of BSL1?

   - different B.C. at $r=R$ (Newton's law of cooling)
   - non-uniform generation function $S_e(r)$.

The B.C. does not affect the shell balance. Newton's law of cooling does not interact with the control volume between r and $(r+\Delta r)$. It doesn't enter the derivation until Eq. 9.2-12. The non-uniform S_e, however, alters the integration of Eq. 9.2-6 to 9.2-7. Eq. 9.2-7 assumes S_e is a constant. Therefore the last equation that applies is Eq. 9.2-6.

(This same issue came up in the Part 1 exam (15 March), problem 1. There, the nonuniform viscosity μ meant that one could not integrate for τ_{rz} assuming μ is constant.)

2. a) Newton's law of cooling specifies $q_r = h\,(T - T_a)$

b) A macroscopic energy balance on the cylinder:

rate of heat lost to surroundings: $(2\pi R L)\,h\,(T-T_a)$

accumulation: $(\pi R^2 L)\rho C_p\, dT/dt$

no generation, no convection

$$-(2\pi R L + 2\pi R^2)\,h(T-T_a) = \pi R^2 L\,\rho C_p\,\frac{dT}{dt}$$

$$\frac{d(T-T_a)}{dt} = -\frac{2\pi R L h}{\pi R^2 L \rho C_p}$$

$$= -\frac{2h}{R\rho C_p}$$

$$\frac{d(T-T_a)}{T-T_a} = -\frac{2h}{R\rho C_p}t$$

$$\ln(T-T_a) = -\frac{2h}{R\rho C_p}t + C_1$$

at $t=0$, $\ln(T-T_a) = 200-20$

$$\frac{T-20}{200-20} = \exp\left(-\frac{2h}{R\rho C_p}t\right) = \exp\left(-\frac{2\cdot 50}{(0.025)(1000)(3??)}t\right)$$

$$= \exp(-2.79\cdot 10^{-3}t)$$

c) There was some confusion among students on the meaning of "gradient," which means derivative with respect to position (dT/dr), not time (dT/dt). As it happens, the maximum in both (dT/dr) and (dT/dt) occurs at $t=0$.

From Fourier's law in the solid,

$$q_r = -k(dT/dr)$$

Since the heat flux at the surface is given by Newton's law of cooling, then at the surface

$$-k\frac{dT}{dr} = h(T-T_a)$$

The maximum in $(T-T_a)$ occurs at $t=0$; it decreases with time.

$$\left.\frac{dT}{dr}\right|_{r=R,\,t=0} = -\frac{q_r}{k} = -\frac{h(T-T_a)}{k} = -\frac{h(180)}{k} = -\frac{50(180)}{35} = 257\ \frac{°K}{m}$$

3. Convective heat transfer is in series with internal conduction. For processes in series, the slow step controls; therefore we want internal conduction to be as fast as possible compared to convective heat transfer to the surface.

Internal conduction depends on $\frac{\alpha t}{R^2}$, with $\alpha = \frac{k}{\rho c_p}$.

For convective heat transfer, leading to a macroscopic energy balance like that in problem 2(b), temperature scales with $\frac{ht}{\rho c_p}$ (see problem 2). Both processes slow down as ρc_p increases, but a larger k makes internal conduction go faster. Therefore the best metal has the larger k. Metal A has 10x larger k; it is the better choice, even though (ρc_p) is larger.

[When I first wrote this problem, I thought the only thing that matters is α. I also accepted answers specifying metal A, because α is about 6x larger, though the complete answer is more complicated.]

4. Because of the perfectly insulated surface, this problem is equivalent to a cube, 0.2 m on a side, where we calculate T at center.

$$\alpha = \frac{k}{\rho c_p} = \frac{23}{7820 \cdot 461} = 6.38 \times 10^{-6}$$

At 5 min, $\frac{\alpha t}{b^2} = \frac{6.38 \cdot 10^{-6}(300)}{(0.1)^2} = 0.19$

From chart for slab, $\frac{T_1 - T}{T_1 - T_0} \doteq 0.77$ at $\frac{y}{b} = 0$

for cube, " $= (0.77)^3 = 0.46$

$$\frac{100 - T}{100 - 0} = 0.46 \ ; \ T = 54°C$$

at 8 min, $\frac{\alpha t}{b^2} = \frac{6.38 \cdot 10^{-6}(480)}{(0.1)^2} = 0.306$

from chart for slab, $\frac{T_1 - T}{T_1 - T_0} = 0.6$ at $y/b = 0$

for cube, " $= (0.6)^3 = 0.29$

$$\frac{T_1 - T}{T_1 - T_0} = 0.29 = \frac{100 - T}{100 - 0} \Rightarrow T = 0.71°C$$

5. For this problem, we need $\frac{T - T_0}{T_1 - T_0}$. We already solved for $\frac{T_1 - T}{T_1 - T_0}$ in problem 4.

At 8 min $\frac{T - T_0}{T_1 - T_0} = \frac{71 - 0}{100 - 0} = 0.71$

5 min " $\frac{54 - 0}{100 - 0} = 0.54$

Dimensionless T is $\frac{T - 0}{100 - 0}$. Second change is opposite to first

$$\frac{T - T_0}{T_1 - T_0} = \left(\frac{T - T_0}{T_1 - T_0}\right)_{8\text{ min}} - \left(\frac{T - T_0}{T_1 - T_0}\right)_{5\text{ min}} = 0.71 - 0.54 = 0.17$$

$$\frac{T - 0}{100 - 0} = 0.17 ; \ T = 17°C$$